The 6th China Collegiate Programming Contest, Finals

Contest Session

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A. Autobiography

Bobo has an **undirected** graph with n vertices and m edges. The vertices are numbered by $1, \dots, n$, and the i-th edge is between the a_i-th and the b_i-th vertex. Plus, the i-th vertex is associated with a character c_i.

Find the number of ways to choose four **distinct** vertices (u, v, w, x) such that

- u and v, v and w, w and x are connected by an edge,
- $c_u = \texttt{b}$, $c_v = \texttt{o}$, $c_w = \texttt{b}$, $c_x = \texttt{o}$.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains two integers n and m.

The second line contains n characters $c_1 \dots c_n$.

For the following m lines, the i-th line contains two integers a_i and b_i.

- $4 \le n \le 2 \times 10^5$
- $0 \le m \le 2 \times 10^5$
- $c_i \in \{\texttt{b}, \texttt{o}\}$ for each $1 \le i \le n$
- $1 \le a_i, b_i \le n$ for each $1 \le i \le m$
- $a_i \ne b_i$ for each $1 \le i \le m$
- $\{a_i, b_i\} \ne \{a_j, b_j\}$ for each $1 \le i < j \le m$
- In each input, the sum of n does not exceed 2×10^5. The sum of m does not exceed 2×10^5.

Output

For each test case, output an integer which denotes the number of ways.

Sample Input

```
5 4
bbobo
1 3
2 3
3 4
4 5
4 6
bobo
1 2
1 3
1 4
2 3
2 4
3 4
4 0
bobo
```

Sample Output

```
2
4
0
```

Note

For the first test case, there are 2 quadrangles $(1, 3, 4, 5)$, $(2, 3, 4, 5)$.

For the second test case, there are 4 quadrangles $(1, 2, 3, 4)$, $(1, 4, 3, 2)$, $(3, 2, 1, 4)$, $(3, 4, 1, 2)$.

For the third test case, there are no valid quadrangles.

B. Algebra

Given three integers n, m, k, find the number of pairs (a, b) where

- $|a|, |b| \leq m$,
- $a, b \in \mathbb{Z}$, i.e., a and b are integers,
- $|S| = k$ where S be the set of rational roots of the equation $x^n + a \cdot x + b = 0$, and $|S|$ is the size of S. In particular, there exists exactly k **distinct rational** numbers x which solve the last equation.

Note: x is a rational number if and only if there exists two integers p and q ($q \neq 0$) where $x = \frac{p}{q}$.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains three integers n, m and k.

- $1 \leq n, m, k \leq 5 \times 10^5$
- In each input, the sum of m does not exceed 5×10^5.

Output

For each test case, output an integer which denotes the number of pairs.

Sample Input

```
2 1 1
2 2 2
3 3 3
```

Sample Output

```
1
7
1
```

Note

For the first test case, only the equation $x^2 = 0$ has one rational root.

For the second test case, each of the following 7 equations has two distinct rational roots.

- $x^2 - 2x = 0$
- $x^2 - x = 0$
- $x^2 - x - 2 = 0$
- $x^2 - 1 = 0$
- $x^2 + x = 0$
- $x^2 + 2x = 0$
- $x^2 + x - 2 = 0$

C. Cryptography

Given three arrays f, g, h of length 2^m, Bobo defines a cryptographic function $\text{enc}(x, y) = (a, b)$ where

- $a = y \oplus g[x \oplus f[y]]$,
- $b = x \oplus f[y] \oplus h[y \oplus g[x \oplus f[y]]]$.

He also has q questions $(a_1, b_1), \dots, (a_q, b_q)$.

For each (a_i, b_i), find a pair of integers (x, y) where $0 \le x, y < 2^m$ and $\text{enc}(x, y) = (a_i, b_i)$. It is guaranteed that for each (a_i, b_i), there exists a **unique** pair (x, y) satisfying the condition.

Note: $\oplus$ denotes the bitwise exclusive-or, i.e., xor.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains two integers m and q.

The second line contains 2^m integers $f[0], \dots, f[2^m - 1]$.

The third line contains 2^m integers $g[0], \dots, g[2^m - 1]$.

The forth line contains 2^m integers $h[0], \dots, h[2^m - 1]$.

For the following q lines, the i-th line contains two integers a_i and b_i.

- $1 \le m \le 16$
- $1 \le q \le 10^5$
- $0 \le f[i], g[i], h[i] < 2^m$ for each $0 \le i < 2^m$
- $0 \le a_i, b_i < 2^m$ for each $1 \le i \le q$
- In each input, the sum of 2^m does not exceed 10^5. The sum of q does not exceed 10^5.

Output

For each question, output two integers which denote the found x and y.

Sample Input

```
2 2
0 1 2 3
1 2 3 0
2 3 0 1
0 1
2 3
1 1
0 0
0 0
0 0
0 0
```

Sample Output

```
3 0
1 2
0 0
```

D. Data Structure

In compute science, a stack s is a data structure maintaining a list of elements with two operations:

- s.`push`(e) appends an element e to the right end of the list,
- s.`pop`$()$ removes the rightmost element in the list and returns the removed element.

For convenience, Bobo denotes the number of elements in the stack s by `size`(s), and the rightmost element by `right`(s).

Bobo has m stacks $s_1, \dots, s_m$. Initially, the stack s_i contains k_i elements $a_{i,1}, \dots, a_{i,k_i}$ where $a_{i,j} \in \{1, \dots, n\}$. Furthermore, for each $e \in \{1, \dots, n\}$, the element e occurs in the m stacks **exactly twice**. Thus, $k_1 + \dots + k_m = 2n$.

A sorting plan of length l consists of l pairs $(f_1, t_1), \dots, (f_l, t_l)$. To execute a sorting plan, for each $i \in \{1, \dots, l\}$ in the increasing order, Bobo performs s_{t_i}.`push`$(s_{f_i}$.`pop`$())$.

A sorting plan is *valid* if the length does not exceed $\lfloor \frac{3n}{2} \rfloor$, and for each $i \in \{1, \dots, l\}$, $1 \le f_i, t_i \le m$, $f_i \ne t_i$. Before the i-th operation,

- `size`$(s_{f_i}) > 0$,
- `size`$(s_{t_i}) < 2$,
- either `size`$(s_{t_i}) = 0$ or `right`$(s_{f_i}) =$ `right`(s_{t_i}).

Also, after the execution of a *valid* sorting plan, each of the m stacks either is empty or contains the two copies of the same element.

Find a *valid* sorting plan, given the initial configuration of the m stacks.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains two integers n and m.

For the next m lines, the i-th line contains an integer k_i, and k_i integers $a_{i,1}, \dots, a_{i,k_i}$.

- $1 \le n \le m \le 2 \times 10^5$
- $0 \le k_i \le 2$ for each $1 \le i \le m$
- $1 \le a_{i,j} \le n$ for each $1 \le i \le m$, $1 \le j \le k_i$
- For each $1 \le e \le n$, there exists exactly two (i, j) where $1 \le j \le k_i$ and $a_{i,j} = e$.
- In each input, the sum of m does not exceed 2×10^5.

Output

For each test case, if there exists a *valid* sorting plan, output an integer l, which denotes the length of the sorting plan. Followed by l lines, the i-th line contains two integers f_i and t_i. Otherwise, output `-1`.

If there are multiple *valid* sorting plans, any of them is considered correct.

Sample Input

```
2 3
2 1 2
2 1 2
0
1 1
2 1 1
3 4
2 1 3
2 2 3
1 1
1 2
```

Sample Output

```
3
1 3
2 3
2 1
0
-1
```

Note

For the first test cases,

- Initially, $s_1 = [1, 2]$, $s_2 = [1, 2]$, $s_3 = [\ ]$.
- After s_3.push(s_1.pop()). $s_1 = [1]$, $s_2 = [1, 2]$, $s_3 = [2]$.
- After s_3.push(s_2.pop()), $s_1 = [1]$, $s_2 = [1]$, $s_3 = [2, 2]$.
- After s_1.push(s_2.pop()), $s_1 = [1, 1]$, $s_2 = [\ ]$, $s_3 = [2, 2]$.

For the second test case, the initial configuration is already sorted.

E. Game Theory

For a string $s_1 \dots s_n$ of n bits (i.e., zeros and ones), Bobo computes the f-value of $s_1 \dots s_n$ by playing the following game.

- If all the bits are zero, the game ends.
- If there are k ones in the bit string, Bobo flips the k-th bit, i.e., s_k.
- The f-value of the bit string is the number of flips Bobo has performed before the game ends.

Formally,

- If $s_1 = \cdots = s_n = 0$, $f(s_1 \dots s_n) = 0$.
- Otherwise, assuming that $k = s_1 + \cdots + s_n$, $f(s_1 \dots s_n) = f(s_1 \dots s_{k-1}\overline{s_k}s_{k+1} \dots s_n) + 1$ where $\overline{c}$ denotes the flip of the bit c such as $\overline{0} = 1$ and $\overline{1} = 0$.

Now, Bobo has a bit string $s_1 \dots s_n$ subjecting to q changes, where the i-th change is to flip all the bits among $s_{l_i} \dots s_{r_i}$ for given l_i, r_i. Find the f-value **modulo** 998244353 of the bit string after each change.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains two integers n and q.

The second line contains n bits $s_1 \dots s_n$.

For the following q lines, the i-th line contains two integers l_i and r_i.

- $1 \leq n \leq 2 \times 10^5$
- $1 \leq q \leq 2 \times 10^5$
- $s_i \in \{0, 1\}$ for each $1 \leq i \leq n$
- $1 \leq l_i \leq r_i \leq n$ for each $1 \leq i \leq q$
- In each input, the sum of n does not exceed 2×10^5. The sum of q does not exceed 2×10^5.

Output

For each change, output an integer which denotes the f-value modulo 998244353.

Sample Input

```
3 2
010
1 2
2 3
5 1
00000
1 5
```

Sample Output

```
1
3
5
```

Note

For the first test case, the string becomes `100` after the first change. $f(\texttt{100}) = f(\texttt{000}) + 1 = 1$. And it becomes `111` after the second change. $f(\texttt{111}) = f(\texttt{110}) + 1 = f(\texttt{100}) + 2 = f(\texttt{000}) + 3 = 3$.

F. Graph Theory

Bobo has an **undirected** graph G with n vertices labeled by $1, \ldots, n$ and n edges. For each $1 \le i \le n$, there is an edge between the vertex i and the vertex $(i \bmod n) + 1$. He also has a list of m pairs $(a_1, b_1), \ldots, (a_m, b_m)$.

Now, Bobo is going to choose an i and remove the edge between the vertex i and the vertex $(i \bmod n) + 1$. Let $\delta_i(u, v)$ be the number of edges on the shortest path between the u-th and the v-th vertex **after the removal**. Choose an i to minimize the maximum among $\delta_i(a_1, b_1), \ldots, \delta_i(a_m, b_m)$.

Formally, find the value of

$$\min_{1 \le i \le n} \left\{ \max_{1 \le j \le m} \delta_i(a_j, b_j) \right\}.$$

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains two integers n and m.

For the following m lines, the i-th line contains two integers a_i and b_i.

- $2 \le n \le 2 \times 10^5$
- $1 \le m \le 2 \times 10^5$
- $1 \le a_i, b_i \le n$ for each $1 \le i \le m$
- In each input, the sum of n does not exeed 2×10^5. The sum of m does not exceed 2×10^5.

Output

For each test case, output an integer which denotes the minimum value.

Sample Input

```
3 2
1 2
2 3
3 2
1 1
2 2
3 3
1 2
2 3
3 1
```

Sample Output

```
1
0
2
```

Note

For the first case,

i	$\delta_i(1,2)$	$\delta_i(2,3)$
1	2	1
2	1	2
3	1	1

Choosing $i = 3$ yields the minimum value 1.

G. Hamilton

Bobo has an $n \times n$ symmetric matrix C consisting of zeros and ones. For a **permutation** $p_1, \dots, p_n$ of $1, \dots, n$, let

$$c_i = \begin{cases} C_{p_i, p_{i+1}} & \text{for } 1 \le i < n \\ C_{p_n, p_1} & \text{for } i = n \end{cases}.$$

The permutation p is *almost monochromatic* if and only if the number of indices i ($1 \le i < n$) where $c_i \ne c_{i+1}$ is **at most one**.

Find an *almost monochromatic* permutation $p_1, \dots, p_n$ for the given matrix C.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains an integer n.

For the following n lines, the i-th line contains n integers $C_{i,1}, \dots, C_{i,n}$.

- $3 \le n \le 2000$
- $C_{i,j} \in \{0, 1\}$ for each $1 \le i, j \le n$
- $C_{i,j} = C_{j,i}$ for each $1 \le i, j \le n$
- $C_{i,i} = 0$ for each $1 \le i \le n$
- In each input, the sum of n does not exceed 2000.

Output

For each test case, if there exists an *almost monochromatic* permutation, output n integers $p_1, \dots, p_n$ which denote the permutation. Otherwise, output `-1`.

If there are multiple *almost monochromatic* permutations, any of them is considered correct.

Sample Input

```
3
001
000
100
4
0000
0000
0000
0000
```

Sample Output

```
3 1 2
2 4 3 1
```

Note

For the first test case, $c_1 = C_{3,1} = 1$, $c_2 = C_{1,2} = 0$, $c_3 = C_{2,3} = 0$. Only when $i = 1$, $c_i \ne c_{i+1}$. Therefore, the permutation $3, 1, 2$ is an almost monochromatic permutation.

H. Nonsense

Given n, x and y, let $f_{n,x,y}(a,b)$ denote the value of

$$\sum_{i=a}^{n-b} \binom{i}{a} x^{i-a} \binom{n-i}{b} y^{n-i-b}.$$

Bobo also has q pairs $(a_1, b_1), \dots, (a_q, b_q)$. Find the value of $f_{n,x,y}(a_1, b_1), \dots, f_{n,x,y}(a_q, b_q)$ **modulo** 998244353.

Note:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}.$$

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains four integers n, x, y and q.

In the following q lines, the i-th line contains two integers a_i and b_i.

- $2 \le n \le 10^9$
- $0 \le x, y < 998244353$
- $1 \le q \le 2 \times 10^5$
- $1 \le a_i, b_i \le 5000$ for each $1 \le i \le q$
- $a_i + b_i \le n$ for each $1 \le i \le q$
- In each input, the sum of $\max(a_1, b_1, \dots, a_q, b_q)$ does not exceed 5000. The sum of q does not exceed 2×10^5.

Output

For each pair, output an integer which denotes the value modulo 998244353.

Sample Input

```
3 1 2 2
1 1
1 2
100 2 3 1
1 1
```

Sample Output

```
6
1
866021789
```

I. Number Theory

Let $o_i = \underbrace{1\dots1}_{i\text{ times}}$ be the number which consists of i ones in its decimal representation.

Bobo has an integer n. Find a sequence of possibly negative integers $(x_1, x_2, \dots,)$ where

- $\sum_{i=1}^{\infty} o_i \cdot x_i = n$,
- $\sum_{i=1}^{\infty} i \cdot |x_i|$ is minimized.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains an integer n.

- $1 \le n < 10^{5000}$
- In each input, the sum of the number of decimal digits of n does not exceed 50000.

Output

For each test case, output an integer which denotes the minimum value of $\sum_{i=1}^{\infty} i \cdot |x_i|$.

Sample Input

```
12
100
998244353
```

Sample Output

```
3
5
76
```

Note

For the first test case, $x_1 = x_2 = 1$, $x_3 = x_4 = \cdots = 0$. The minimum value is $1 \times 1 + 2 \times 1 = 3$.

For the second test case, $x_1 = 0$, $x_2 = -1$, $x_3 = 1$, $x_4 = x_5 = \cdots = 0$. The minimum value is $2 \times 1 + 3 \times 1 = 5$.

J. Permutation Pattern

A sequence $a_1, \ldots, a_m$ of m distinct numbers is called *without 231* if there is **no** triples (i, j, k) where $1 \le i < j < k \le m$ and $a_k < a_i < a_j$.

Bobo has a permutation $p_1, \ldots, p_n$ of $1, \ldots, n$, and he can remove some (possibly none, but not all) elements from the permutation. Find the number of sequences without 231 among $(2^n - 1)$ resulting permutations.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains an integer n.

The second line contains n integers $p_1, \ldots, p_n$.

- $1 \le n \le 50$
- $1 \le p_i \le n$ for each $1 \le i \le n$
- In each input, the sum of n does not exceed 500.

Output

For each test case, output an integer which denotes the number of sequences.

Sample Input

```
2
2 1
3
1 2 3
4
2 3 4 1
```

Sample Output

```
3
7
11
```

K. Stringology

For a string $u = u_1 \dots u_n$, Bobo denotes the prefix $u_1 \dots u_i$ by $\mathrm{pre}(u, i)$. Similarly, he denotes the suffix $u_{n-i+1} \dots u_n$ by $\mathrm{suf}(u, i)$. In particular, $\mathrm{pre}(u, 0)$ and $\mathrm{suf}(u, 0)$ are empty strings.

For two strings $u = u_1 \dots u_n$ and $v = v_1 \dots v_m$, Bobo denotes the concatenation $u_1 \dots u_n v_1 \dots v_m$ by $u + v$. Also,

$$\mathrm{presuf}(u, v) = \max\{i \mid i < n \text{ and } i \leq m \text{ and } \mathrm{pre}(u, i) = \mathrm{suf}(v, i)\}.$$

Given two strings $s = s_1 \dots s_n$ and $t = t_1 \dots t_m$, let $f(i) = \mathrm{presuf}(s, \mathrm{pre}(s, i) + t)$. Find the value of $f(0), \dots, f(n-1)$.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains a string $s_1 \dots s_n$.

The second line contains a string $t_1 \dots t_m$.

- $1 \leq n, m \leq 10^6$
- $s_i \in \{a, \dots, z\}$ for each $1 \leq i \leq n$
- $t_i \in \{a, \dots, z\}$ for each $1 \leq i \leq m$
- In each input, the sum of $\max(n, m) \leq 10^6$.

Output

For each test case, output n integers which denote $f(0), \dots, f(n-1)$.

Sample Input

```
aaa
a
ababa
a
ab
cd
```

Sample Output

```
1 2 2
1 1 3 1 3
0 0
```

Note

For the second case, $f(4) = \mathrm{presuf}(s, \mathrm{pre}(s, 4) + t) = \mathrm{presuf}(\texttt{ababa}, \texttt{abab} + \texttt{a}) = \mathrm{presuf}(\texttt{ababa}, \texttt{ababa})$.

i	$\mathrm{pre}(\texttt{ababa}, i)$	$\mathrm{suf}(\texttt{ababa}, i)$
0	(an empty string)	(an empty string)
1	a	a
2	ab	ba
3	aba	aba
4	abab	baba

Therefore, $f(4) = 3$.

L. 2D Geometry

There are n **distinct** points on a 2-dimension plane. The coordinates of the i-th point is (x_i, y_i).

If there are three points A, B and C which form a triangle ABC with **positive area**, Bobo can remove them simultaneously from the plane. Also, if there are multiple triangles with positive area, Bobo can choose to remove any of them. Find the minimum number of points left on the plane if he can perform the operation for **any number of times**.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

The first line contains an integer n.

For the following n lines, the i-th line contains two integers x_i and y_i.

- $1 \le n \le 2 \times 10^5$
- $0 \le x_i, y_i \le 10^9$ for each $1 \le i \le n$
- $(x_i, y_i) \ne (x_j, y_j)$ for each $1 \le i < j \le n$
- In each input, the sum of n does not exceed 2×10^5.

Output

For each test case, output an integer which denotes the minimum number of points left.

Sample Input

```
3
0 0
0 1
0 2
3
0 0
0 1
1 0
6
0 0
0 1
0 2
0 3
1 1
1 2
```

Sample Output

```
3
0
0
```

Note

For the third test case, if Bobo chooses to remove the triangle $\{(0,1),(1,1),(1,2)\}$ first, there will be no other triangles to remove. Alternatively, Bobo can remove the triangle $\{(0,0),(0,1),(1,1)\}$ first and then $\{(0,2),(0,3),(1,2)\}$.

M. 3D Geometry

An axis-aligned tetrahedron (also known as triangular pyramid) $DABC$ is a convex polyhedron in three dimension with vertices

- $D : (x_1, y_1, z_1)$,
- $A : (x_2, y_1, z_1)$,
- $B : (x_1, y_2, z_1)$,
- $C : (x_1, y_1, z_2)$.

Also, an axis-aligned cube $PQRSDEFG$ is a convex polyhedron with vertices

- $P : (x_3, y_3, z_3)$,
- $Q : (x_3, y_3, z_4)$,
- $R : (x_3, y_4, z_3)$,
- $S : (x_3, y_4, z_4)$,
- $D : (x_4, y_3, z_3)$,
- $E : (x_4, y_3, z_4)$,
- $F : (x_4, y_4, z_3)$,
- $G : (x_4, y_4, z_4)$.

Given an axis-aligned tetrahedron $DABC$ and an axis-aligned cube $PQRSDEFG$, find the volume of their intersection.

Input

The input consists of several test cases terminated by end-of-file. For each test case,

There are 4 lines, and the i-th line contains three integers x_i, y_i, and z_i.

- $-500 \le x_i, y_i, z_i \le 500$ for each $1 \le i \le 4$
- $x_1 \ne x_2$, $y_1 \ne y_2$, $z_1 \ne z_2$
- $x_3 \ne x_4$, $y_3 \ne y_4$, $z_3 \ne z_4$
- In each input, the number of test cases does not exceed 10^5.

Output

For each test case, output a float which denotes the volume.

Your answer is considered correct if its *absolute* or *relative error* doesn't exceed 10^{-6}.

Sample Input

```
0 0 0
1 1 1
0 0 0
1 1 1
0 0 0
2 2 2
0 0 0
1 1 1
0 2 0
2 0 2
1 0 1
0 1 0
```

Sample Output

```
0.166666667
0.833333333
0.166666667
```